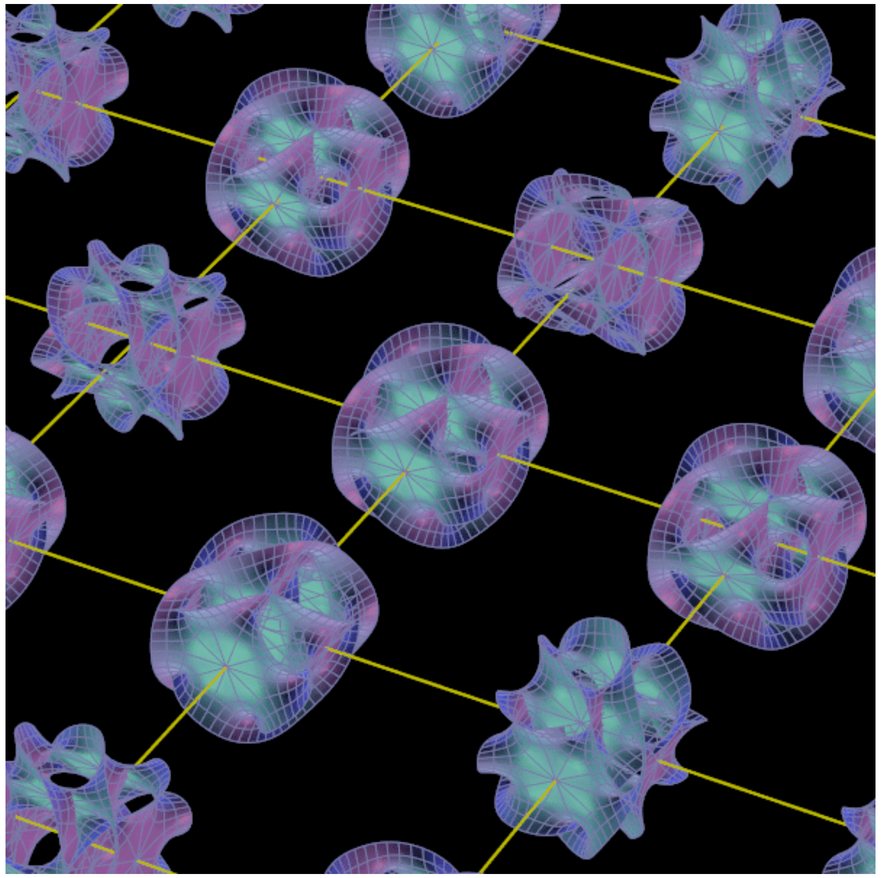
Extra Dimensions in Physics?



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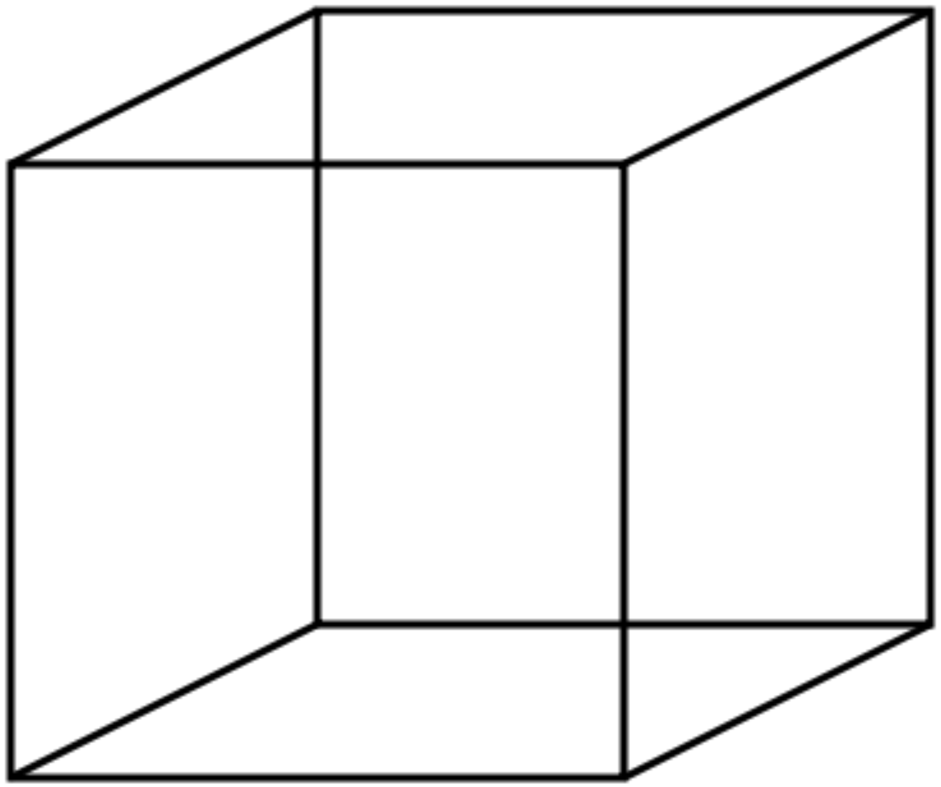
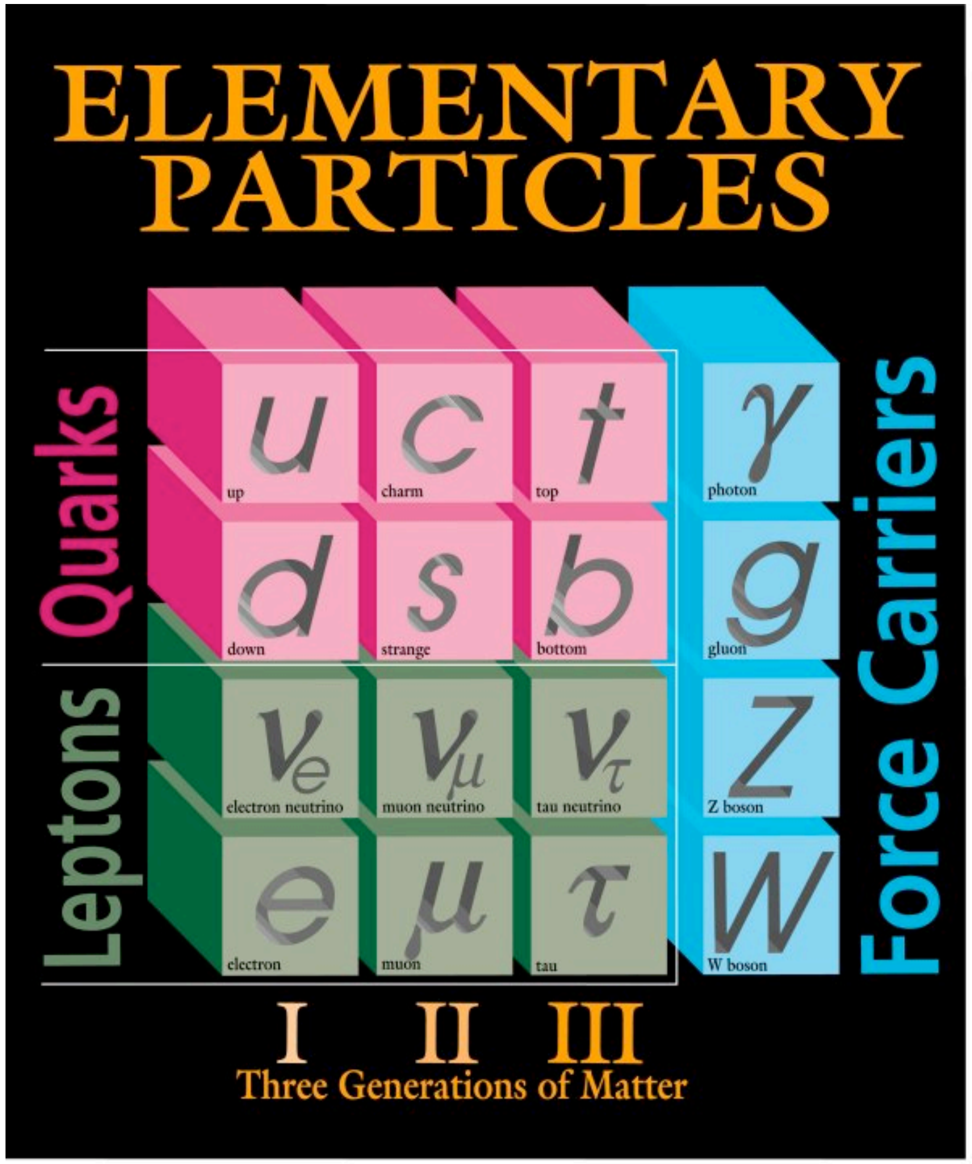
One of the few bits of fundamental physics that becomes "obvious" to most of us in childhood: our playing field consists of three spatial dimensions, and time.



This seemingly obvious truth is no longer believed by many theoretical physicists.

There are a few reasons for this.

Consider today's "periodic table" of fundamental particles:



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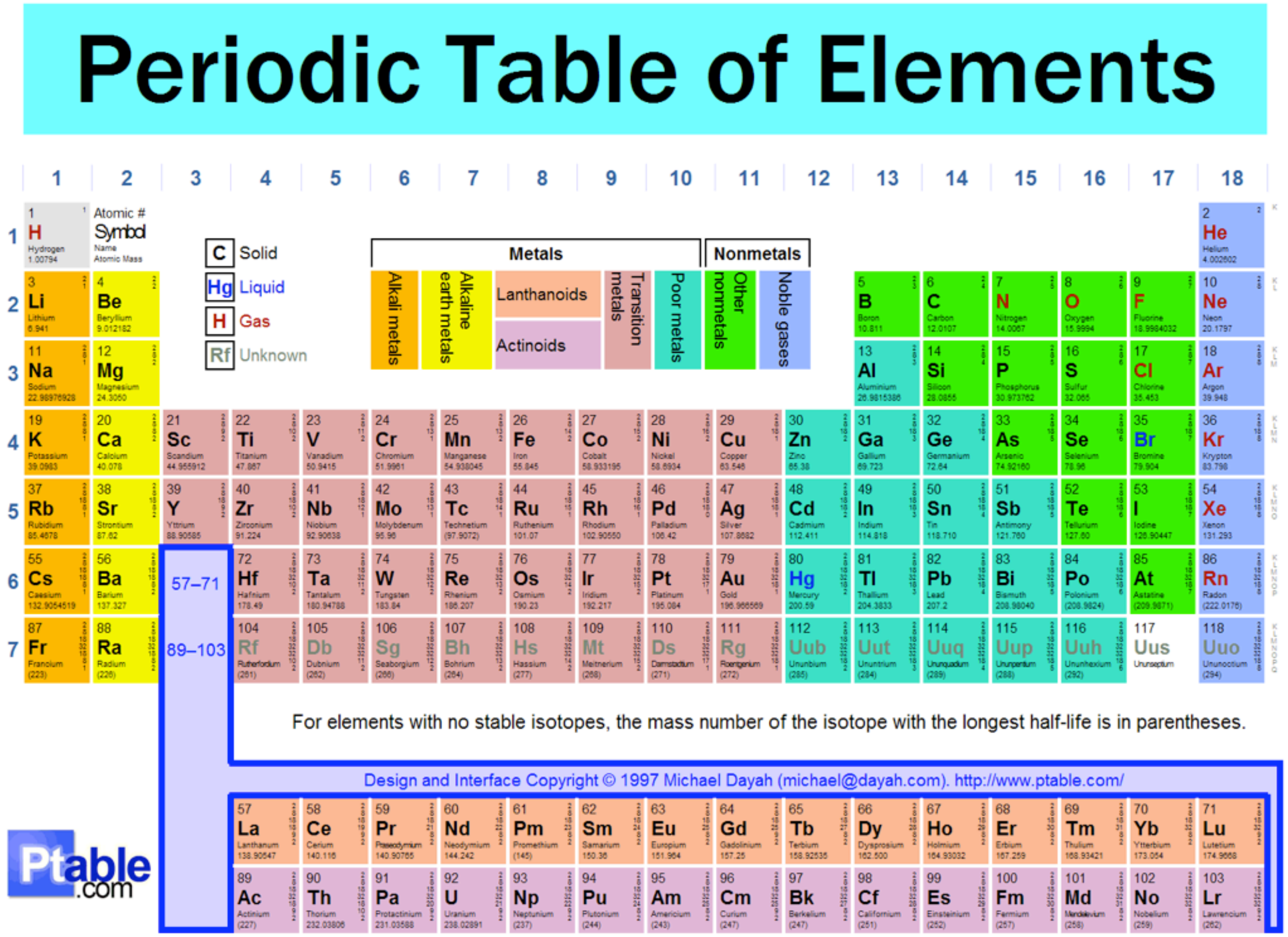
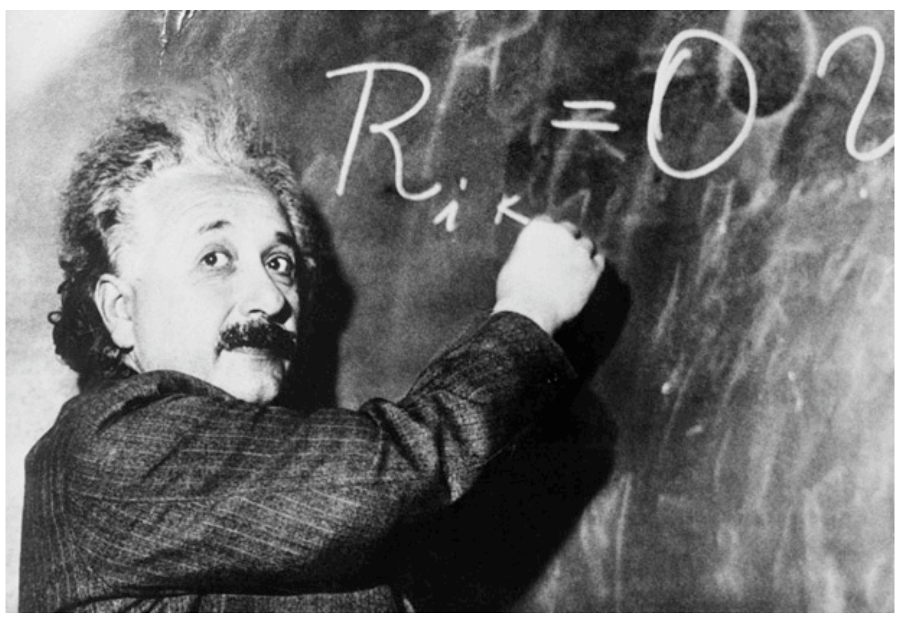
Too many sorts! We should try to unify them.

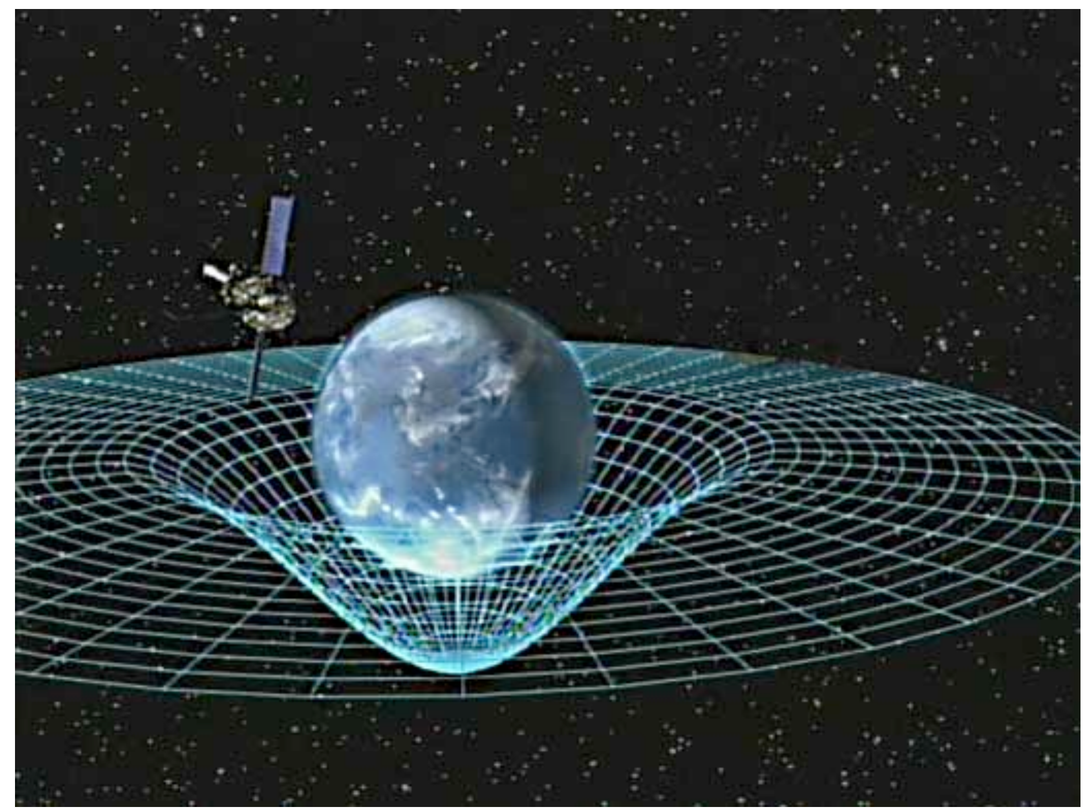
For instance, a similar table in chemistry:



is elegantly explained just by the quantum mechanics of electrons, protons, and neutrons.

In the case at hand, the explanation will likely involve more than just "compositeness." This is because one of the elementary particles, the "graviton," is deeply tied via Einstein's theory to the geometry of space-time itself:





In gravity, matter tells space how to curve,
and curved space-time tells matter how to move.

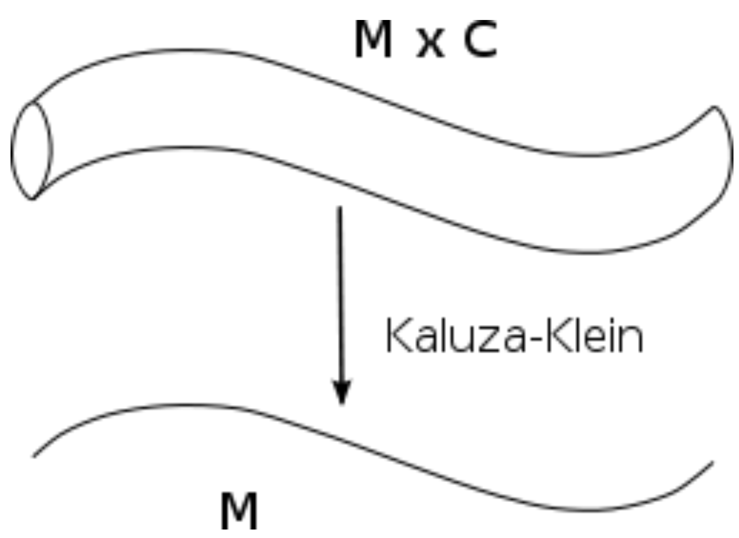
Thus, any theory "unifying" the graviton with other types of force carriers, must ipso facto be a modified theory of space-time geometry.

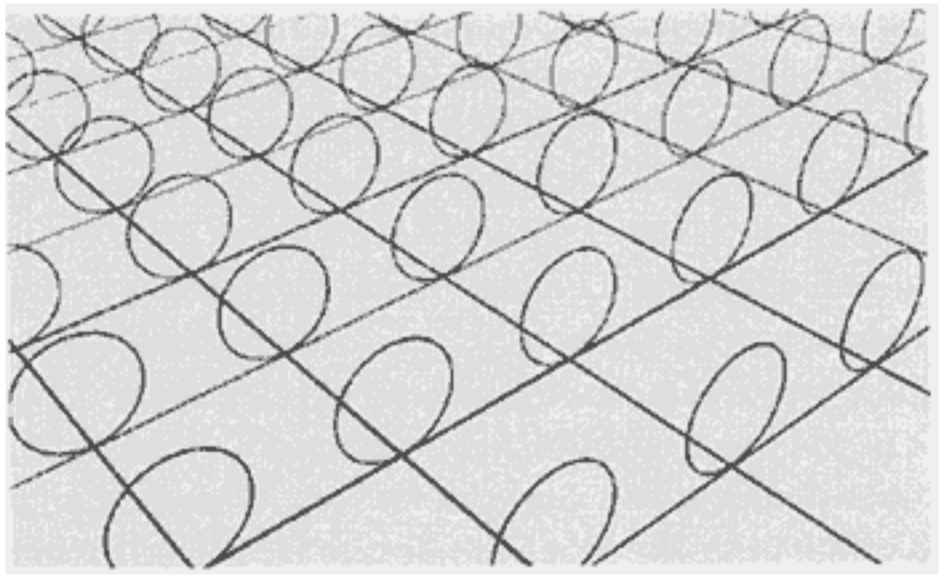
The oldest idea

The oldest idea of unification along these lines dates to ~ 1920. At that time, the known forces were gravity and electromagnetism.

Theodor Kaluza and Oskar Klein realized that these could arise from a theory with just gravity in one higher dimension.

Their idea was that above each point of our three-dimensional space, there lies also a circle:





If the circle is sufficiently small (by experiment today, less than ~.01 mm in size), then we would not have directly detected it yet.

We should then take the 5d physics, reduce it on the circle (which we haven't seen yet), and ask, what would the consequent 4d world look like?

In Einstein's theory, the dynamics of space-time is encoded in a "metric tensor," and the graviton is a small fluctuation of this tensor. For instance the distance between two points in 3d space is given by:

$$\Delta \mathbf{x} = (\Delta x, \Delta y, \Delta z)$$

$$(\text{distance})^2 = g_{ij} \Delta x_i \Delta x_j$$

So the (spatial) metric is a 3x3 (symmetric) matrix.

Now, imagine an analogue of Einstein gravity in 4+1 space-time dimensions. There would be a new coordinate displacement separating two points, the separation "w" on the new circle:

$$\Delta\mathbf{x} = (\Delta x, \Delta y, \Delta z, \Delta w)$$

$$(\text{distance})^2 = g_{ab}\Delta x_a \Delta x_b$$

The new matrix is 4x4 and has 4 extra components relative to Einstein gravity in three spatial dimensions.

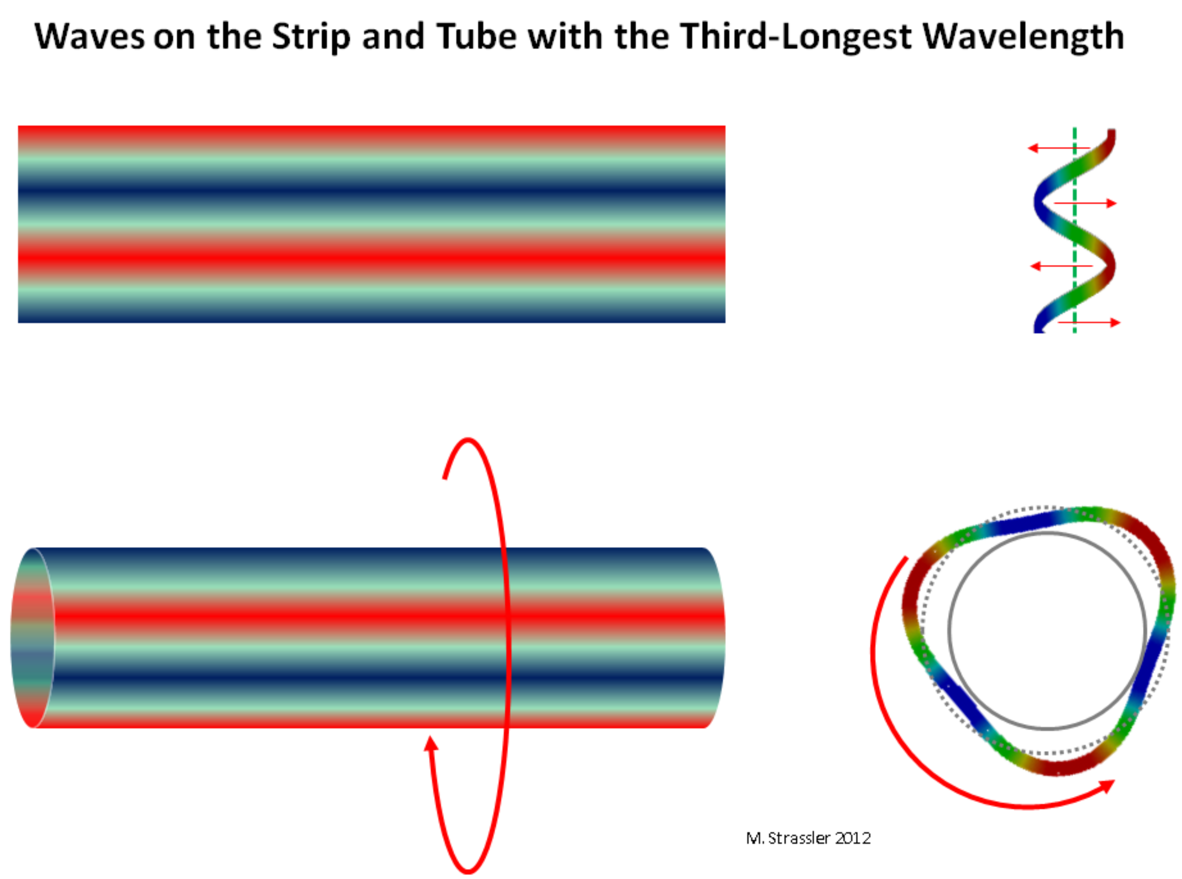
* We can interpret one of these as parametrising the "radius" of the circle above each point in space.

* The other three have precisely the right mathematical structure to give rise to an electromagnetic field in the lower dimensional theory!

$$\hat{g}_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} g_{\mu\nu} - \phi A_\mu A_\nu & -\phi A_\mu \\ -\sigma A_\nu & -\phi \end{pmatrix}$$

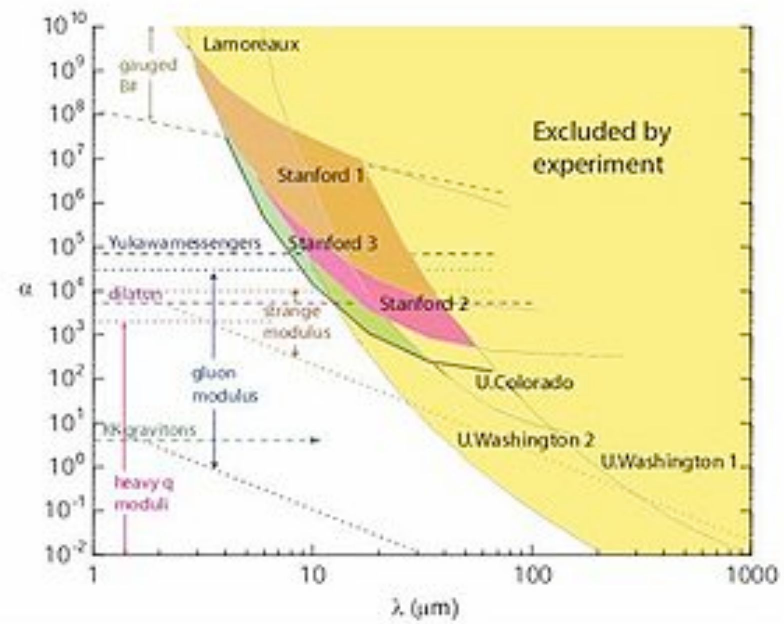
In such a theory, the 4d Coulomb interaction between charged particles will be modified. The 4d photon arises from the "zero momentum mode" of the extra metric components in the 5th circle.

But there are also excited "Kaluza-Klein" cousins of the photon, whose momentum on the circle makes them massive in 4d:



Exchange of these "massive photons" would modify the force between charges in the real world.

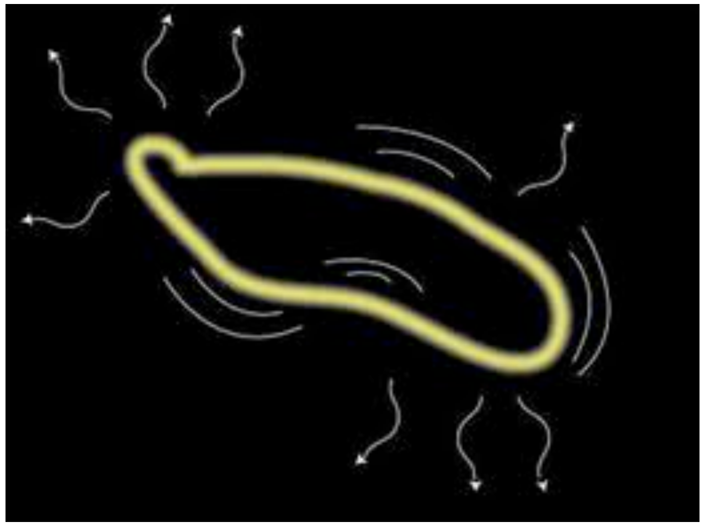
Experiment has not revealed any such modifications as yet:



Modern incarnation of extra dimensions

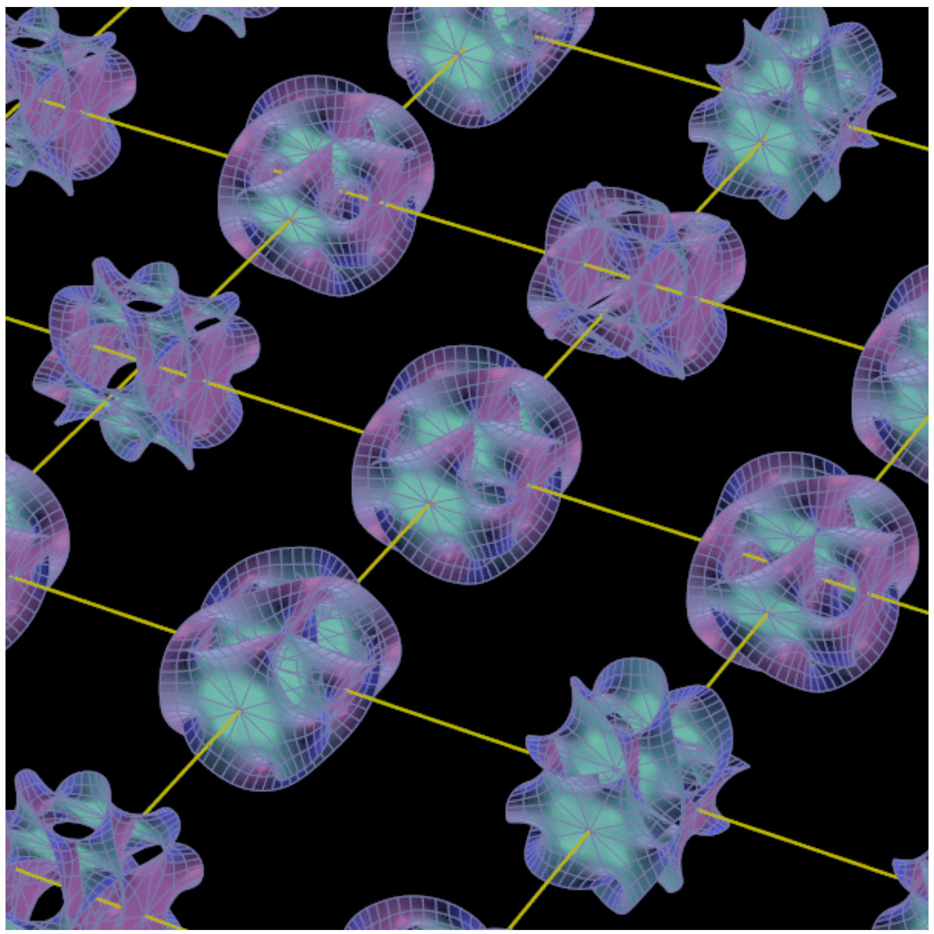
Since the time of Kaluza and Klein, we've learned about more fundamental interactions, and many new particles.

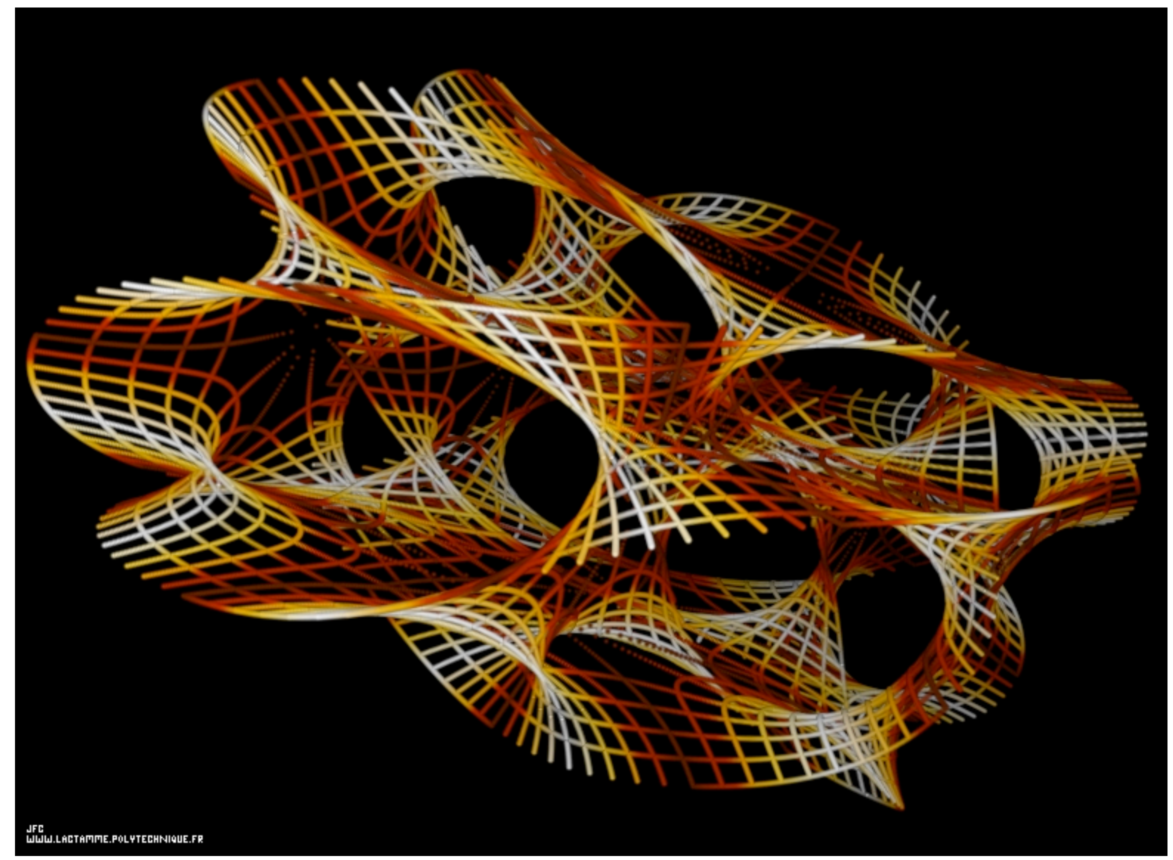
Our most promising way of making Einstein's theory consistent with quantum mechanics, replaces elementary particles with vibrating "superstrings":



Mathematical consistency of these theories requires the existence of extra space-time dimensions; 6 new spatial dimensions are preferred in the simplest constructions.

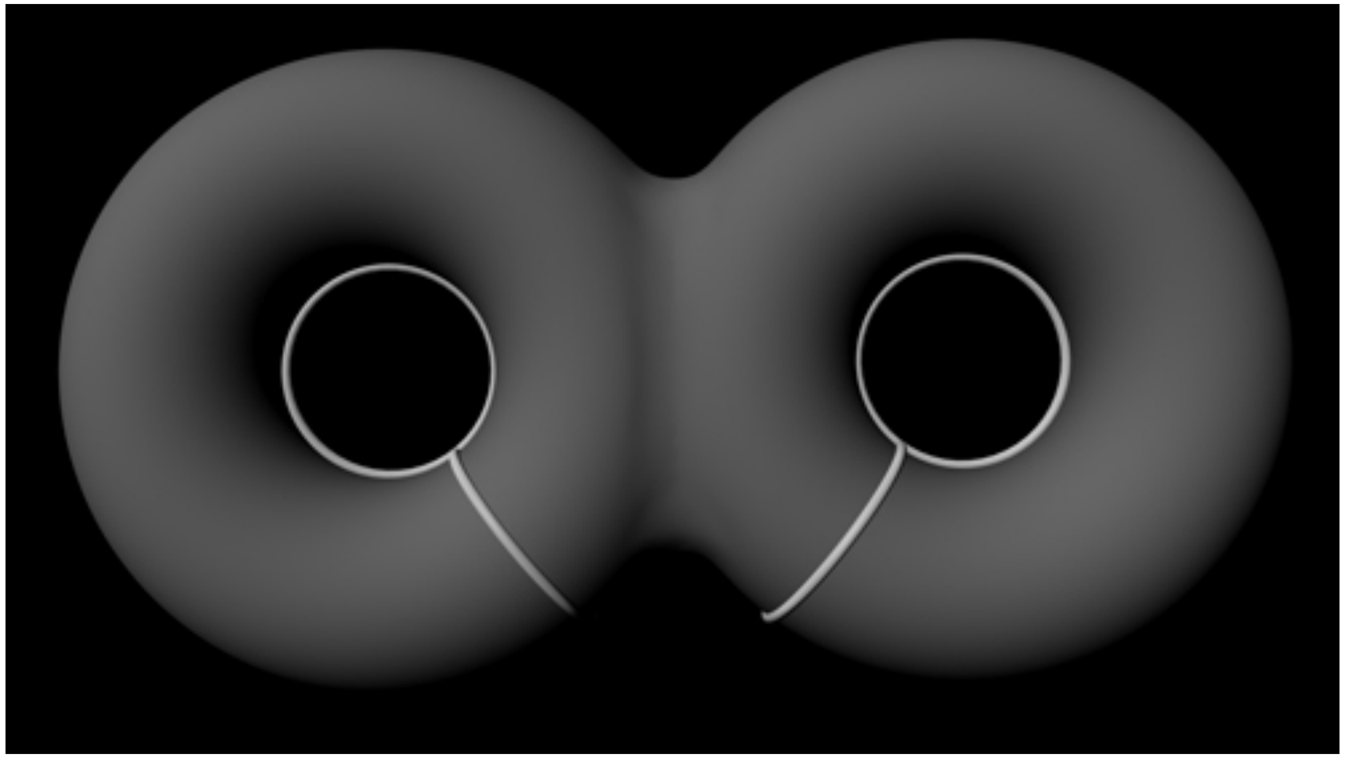
The result is a rich generalization of the original Kaluza-Klein picture, where above each point in our space lies a six-dimensional "Calabi-Yau" manifold:





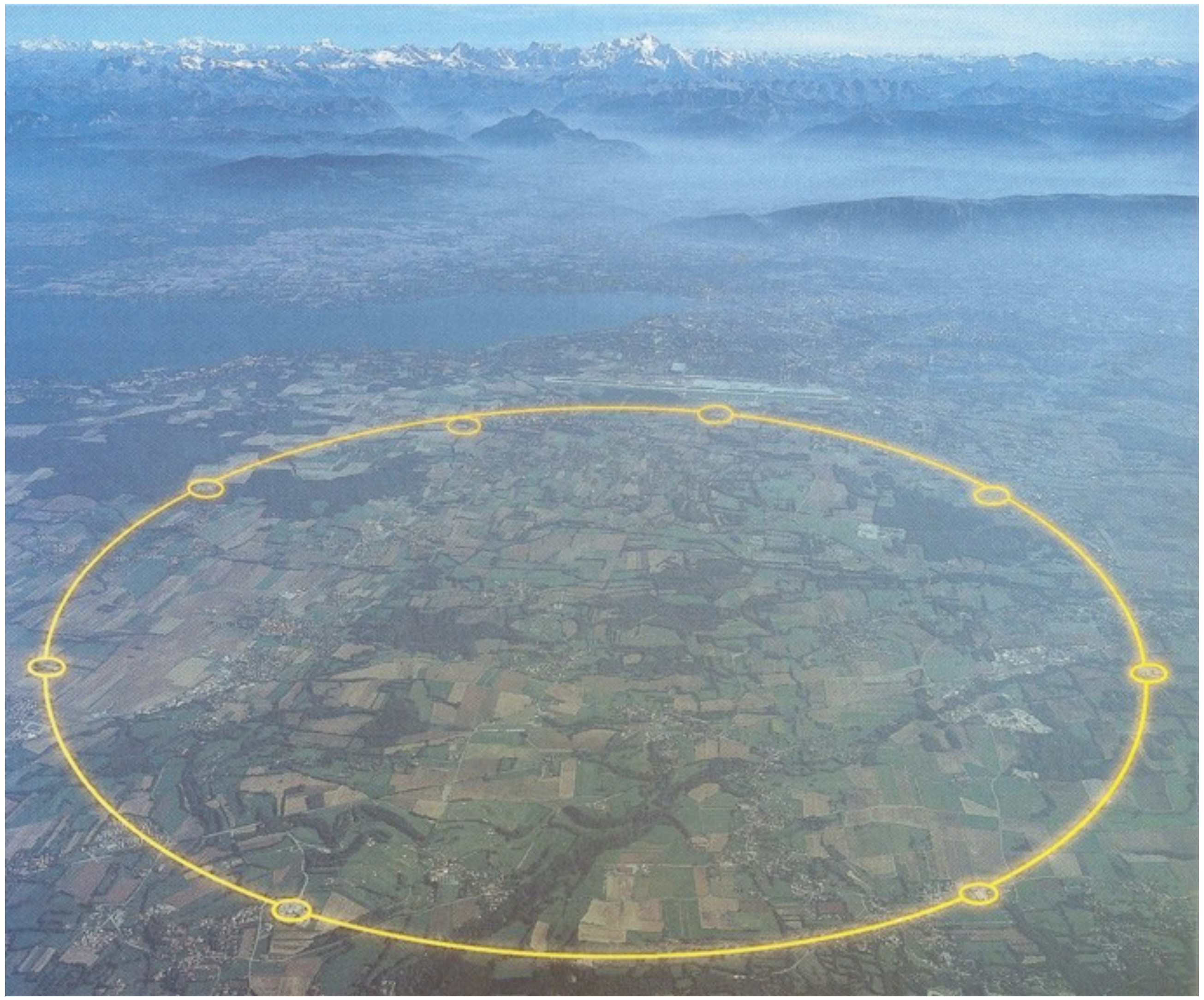
Thousands of possible such spaces with distinct topology are now known. They are rather beautiful.

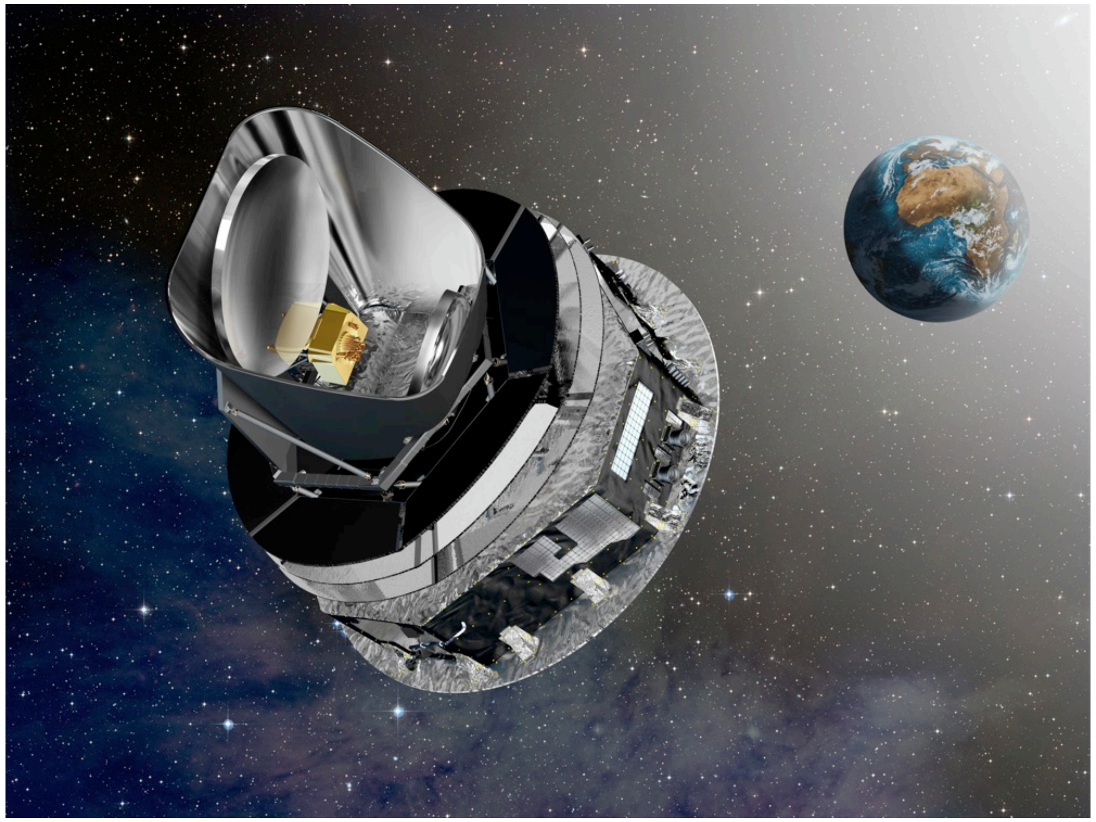
The richer spectrum of extra dimensions and topologies gives string theorists the opportunity to explain the richer set of currently known forces and "elementary" particles, in terms of wrapped strings and branes:



Wrapping topologically distinct cycles yields different flavors of lower-dimensional particles.

We do not know whether this geometrization of physics via extra dimensions is true of our world or not. But we aim to find out!





Thanks for your attention!

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