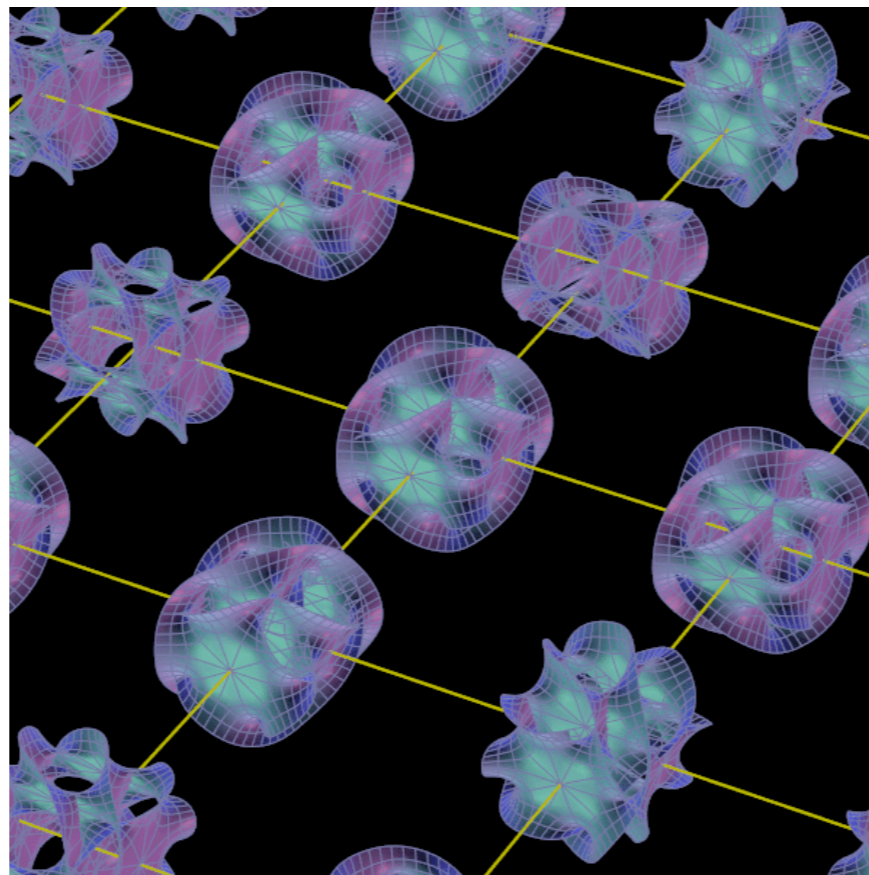
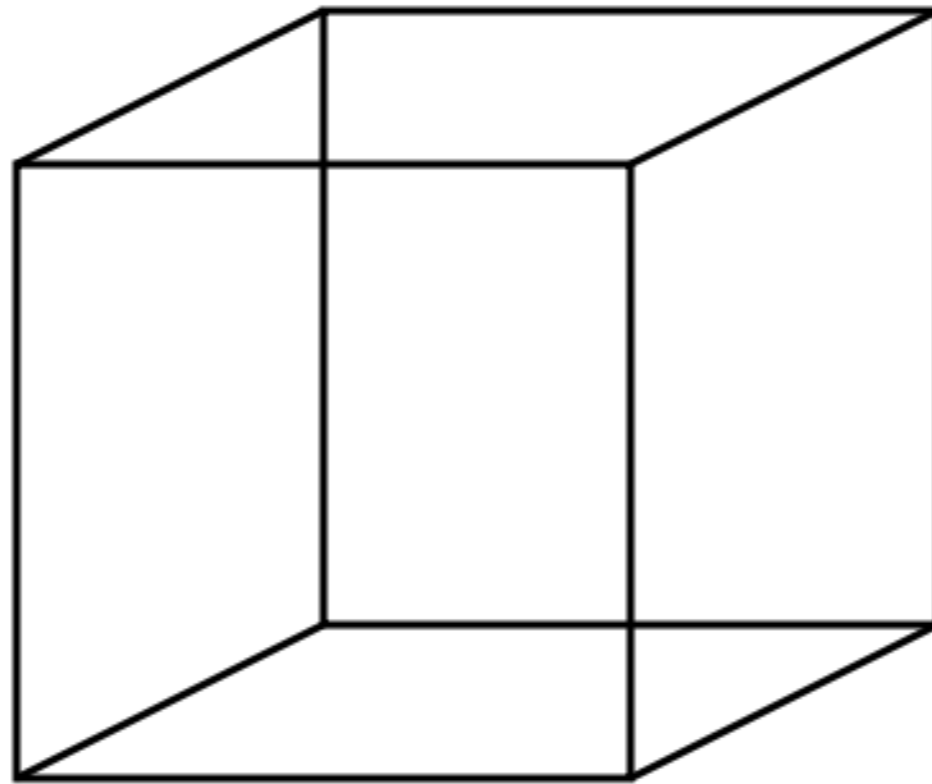


Extra Dimensions in Physics?



Shamit Kachru
Stanford University

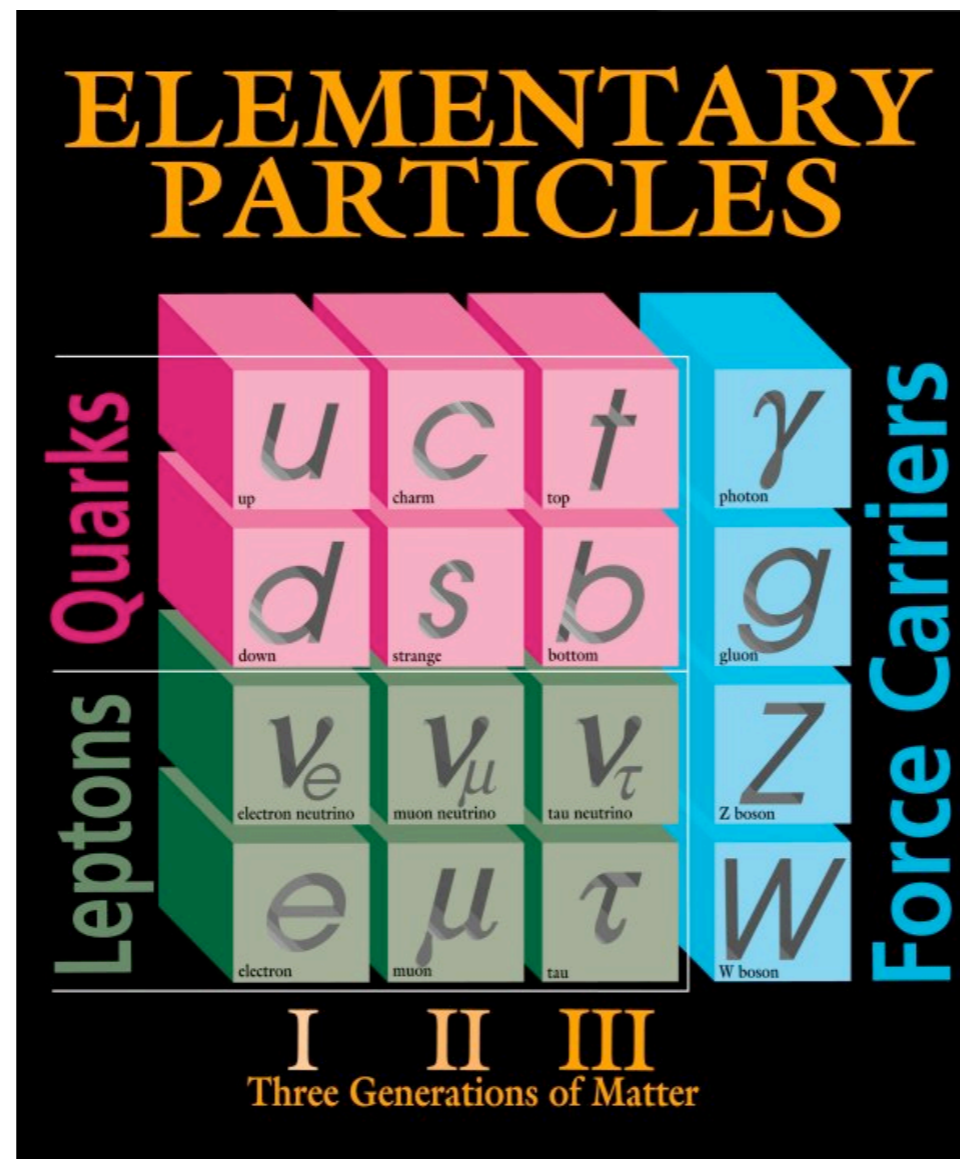
One of the few bits of fundamental physics that becomes “obvious” to most of us in childhood: our playing field consists of three spatial dimensions, and time.



This seemingly obvious truth is no longer believed by many theoretical physicists.

There are a few reasons for this.

Consider today's "periodic table" of fundamental particles:



Fermilab 95-759

Too many sorts! We should try to unify them.

For instance, a similar table in chemistry:

Periodic Table of Elements

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

1 **H** Atomic #
Hydrogen 1.00794 Symbol
Name Atomic Mass

2 **He**
Helium 4.002602

3 **Li**
Lithium 6.941

4 **Be**
Beryllium 9.012182

5 **B**
Boron 10.811

6 **C**
Carbon 12.0107

7 **N**
Nitrogen 14.0067

8 **O**
Oxygen 15.9994

9 **F**
Fluorine 18.9984032

10 **Ne**
Neon 20.1797

11 **Na**
Sodium 22.98976928

12 **Mg**
Magnesium 24.3050

13 **Al**
Aluminum 26.9815386

14 **Si**
Silicon 28.0855

15 **P**
Phosphorus 30.973762

16 **S**
Sulfur 32.065

17 **Cl**
Chlorine 35.453

18 **Ar**
Argon 39.948

19 **K**
Potassium 39.0983

20 **Ca**
Calcium 40.078

21 **Sc**
Scandium 44.955912

22 **Ti**
Titanium 47.887

23 **V**
Vanadium 50.9415

24 **Cr**
Chromium 51.9961

25 **Mn**
Manganese 54.938045

26 **Fe**
Iron 55.845

27 **Co**
Cobalt 58.933195

28 **Ni**
Nickel 58.6934

29 **Cu**
Copper 63.546

30 **Zn**
Zinc 65.38

31 **Ga**
Gallium 69.723

32 **Ge**
Germanium 72.64

33 **As**
Arsenic 74.92160

34 **Se**
Selenium 78.96

35 **Br**
Bromine 79.904

36 **Kr**
Krypton 83.798

37 **Rb**
Rubidium 85.4678

38 **Sr**
Strontium 87.62

39 **Y**
Yttrium 88.90585

40 **Zr**
Zirconium 91.224

41 **Nb**
Niobium 92.90638

42 **Mo**
Molybdenum 95.96

43 **Tc**
Technetium (97.9072)

44 **Ru**
Ruthenium 101.07

45 **Rh**
Rhodium 102.90550

46 **Pd**
Palladium 106.42

47 **Ag**
Silver 107.8682

48 **Cd**
Cadmium 112.411

49 **In**
Indium 114.818

50 **Sn**
Tin 118.710

51 **Sb**
Antimony 121.760

52 **Te**
Tellurium 127.60

53 **I**
Iodine 126.90447

54 **Xe**
Xenon 131.293

55 **Cs**
Cesium 132.9054519

56 **Ba**
Barium 137.327

57-71 **Lanthanoids**

72 **Hf**
Hafnium 178.49

73 **Ta**
Tantalum 180.94788

74 **W**
Tungsten 183.84

75 **Re**
Rhenium 186.207

76 **Os**
Osmium 190.23

77 **Ir**
Iridium 192.217

78 **Pt**
Platinum 195.084

79 **Au**
Gold 196.966569

80 **Hg**
Mercury 200.59

81 **Tl**
Thallium 204.3833

82 **Pb**
Lead 207.2

83 **Bi**
Bismuth 208.98040

84 **Po**
Polonium (209)

85 **At**
Astatine (210)

86 **Rn**
Radon (222)

87 **Fr**
Francium (223)

88 **Ra**
Radium (226)

89-103 **Actinoids**

104 **Rf**
Rutherfordium (261)

105 **Db**
Dubnium (262)

106 **Sg**
Seaborgium (266)

107 **Bh**
Bohrium (264)

108 **Hs**
Hassium (277)

109 **Mt**
Meitnerium (268)

110 **Ds**
Darmstadtium (271)

111 **Rg**
Roentgenium (272)

112 **Uub**
Ununbium (285)

113 **Uut**
Ununtrium (284)

114 **Uuq**
Ununquadium (289)

115 **Uup**
Ununpentium (288)

116 **Uuh**
Ununhexium (282)

117 **Uus**
Ununseptium

118 **Uuo**
Ununoctium (284)

57 **La**
Lanthanum 138.90547

58 **Ce**
Cerium 140.116

59 **Pr**
Praseodymium 140.90765

60 **Nd**
Neodymium 144.242

61 **Pm**
Promethium (145)

62 **Sm**
Samarium 150.36

63 **Eu**
Europium 151.964

64 **Gd**
Gadolinium 157.25

65 **Tb**
Terbium 158.92535

66 **Dy**
Dysprosium 162.500

67 **Ho**
Holmium 164.93032

68 **Er**
Erbium 167.259

69 **Tm**
Thulium 168.93421

70 **Yb**
Ytterbium 173.054

71 **Lu**
Lutetium 174.967

89 **Ac**
Actinium (227)

90 **Th**
Thorium 232.03806

91 **Pa**
Protactinium 231.03688

92 **U**
Uranium 238.02891

93 **Np**
Neptunium (237)

94 **Pu**
Plutonium (244)

95 **Am**
Americium (243)

96 **Cm**
Curium (247)

97 **Bk**
Berkelium (247)

98 **Cf**
Californium (251)

99 **Es**
Einsteinium (252)

100 **Fm**
Fermium (257)

101 **Md**
Mendelevium (258)

102 **No**
Nobelium (259)

103 **Lr**
Lawrencium (262)

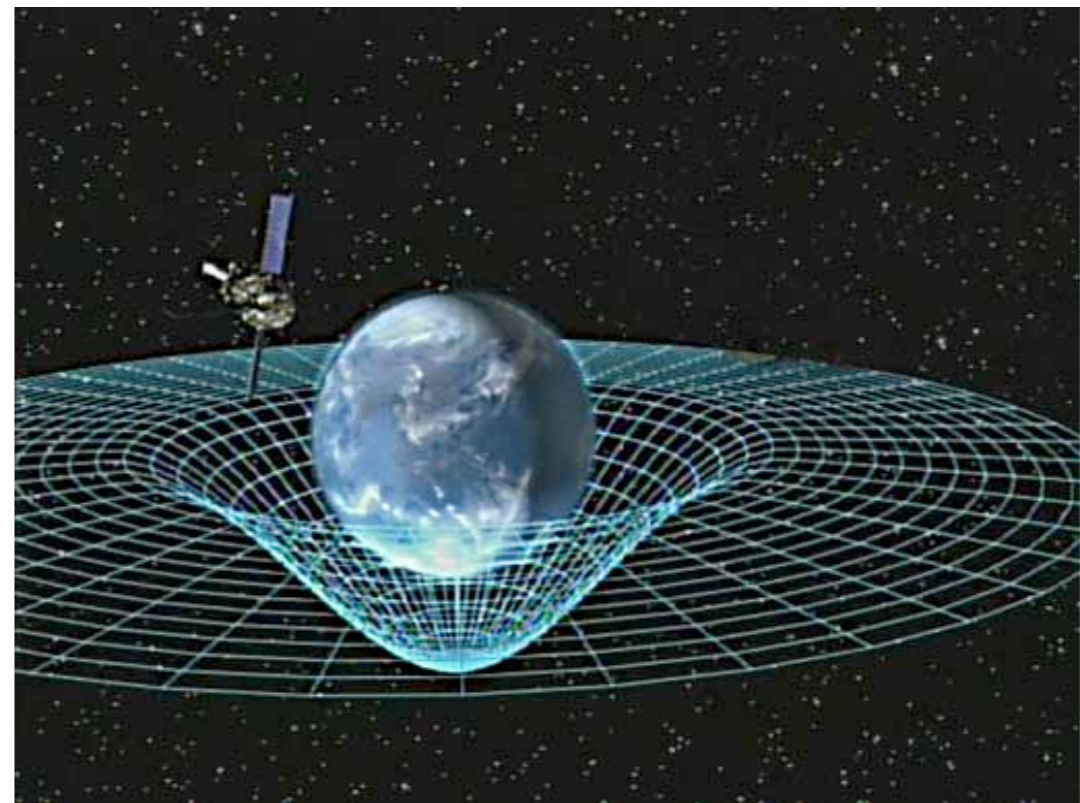
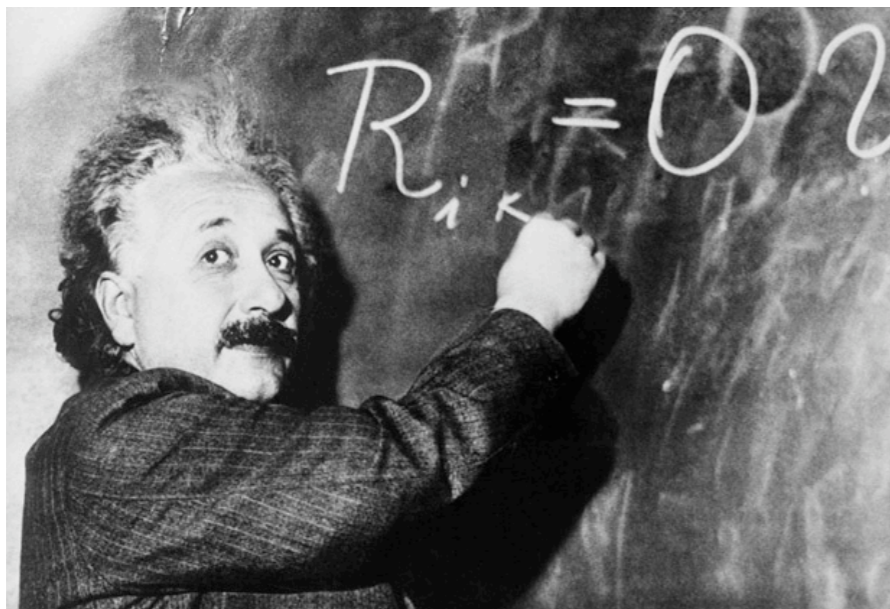
For elements with no stable isotopes, the mass number of the isotope with the longest half-life is in parentheses.

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is elegantly explained just by the quantum mechanics of electrons, protons, and neutrons.

In the case at hand, the explanation will likely involve more than just “compositeness.” This is because one of the elementary particles, the “graviton,” is deeply tied via Einstein’s theory to the geometry of space-time itself:



In gravity, matter tells space how to curve, and curved space-time tells matter how to move.

Thus, any theory “unifying” the graviton with other types of force carriers, must ipso facto be a modified theory of space-time geometry.

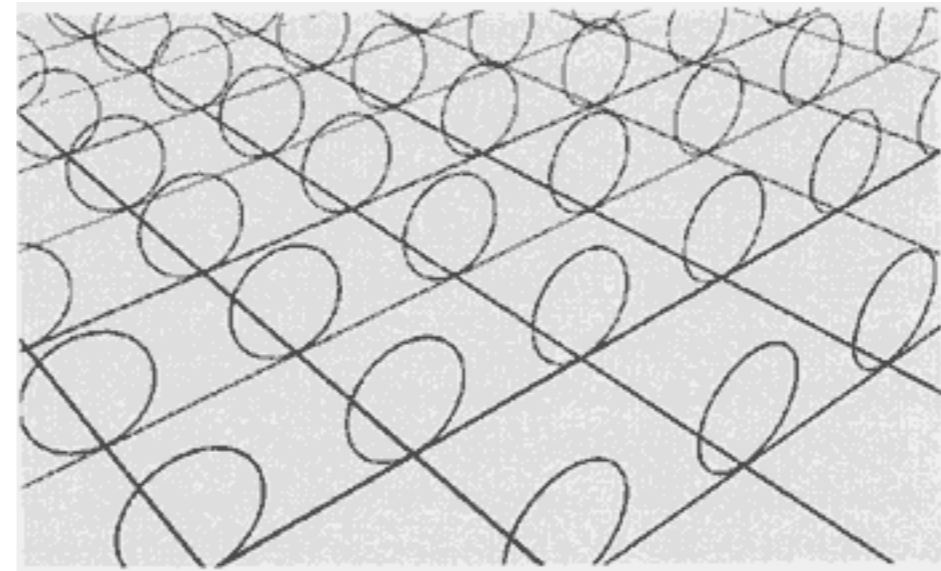
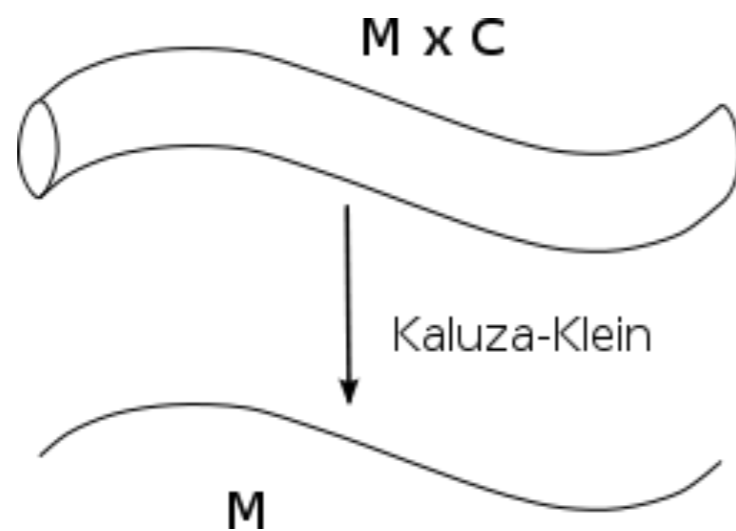
The oldest idea

The oldest idea of unification along these lines dates to ~ 1920. At that time, the known forces were **gravity** and **electromagnetism**.



Theodor Kaluza and Oskar Klein realized that these could arise from a theory with just gravity in one higher dimension.

Their idea was that above each point of our three-dimensional space, there lies also a circle:



If the circle is sufficiently small (by experiment today, less than ~ 0.1 mm in size), then **we would not have directly detected it yet.**

We should then take the 5d physics, reduce it on the circle (which we haven't seen yet), and ask, what would the consequent 4d world look like?

In Einstein's theory, the dynamics of space-time is encoded in a "metric tensor," and the graviton is a small fluctuation of this tensor. For instance the distance between two points in 3d space is given by:

$$\Delta \mathbf{x} = (\Delta x, \Delta y, \Delta z)$$

$$(\text{distance})^2 = g_{ij} \Delta x_i \Delta x_j$$

So the (spatial) metric is a 3x3 (symmetric) matrix.

Now, imagine an analogue of Einstein gravity in 4+1 space-time dimensions. There would be a new coordinate displacement separating two points, the separation “w” on the new circle:

$$\Delta \mathbf{x} = (\Delta x, \Delta y, \Delta z, \Delta w)$$

$$(\text{distance})^2 = g_{ab} \Delta x_a \Delta x_b$$

The new matrix is 4x4 and has 4 extra components relative to Einstein gravity in three spatial dimensions.

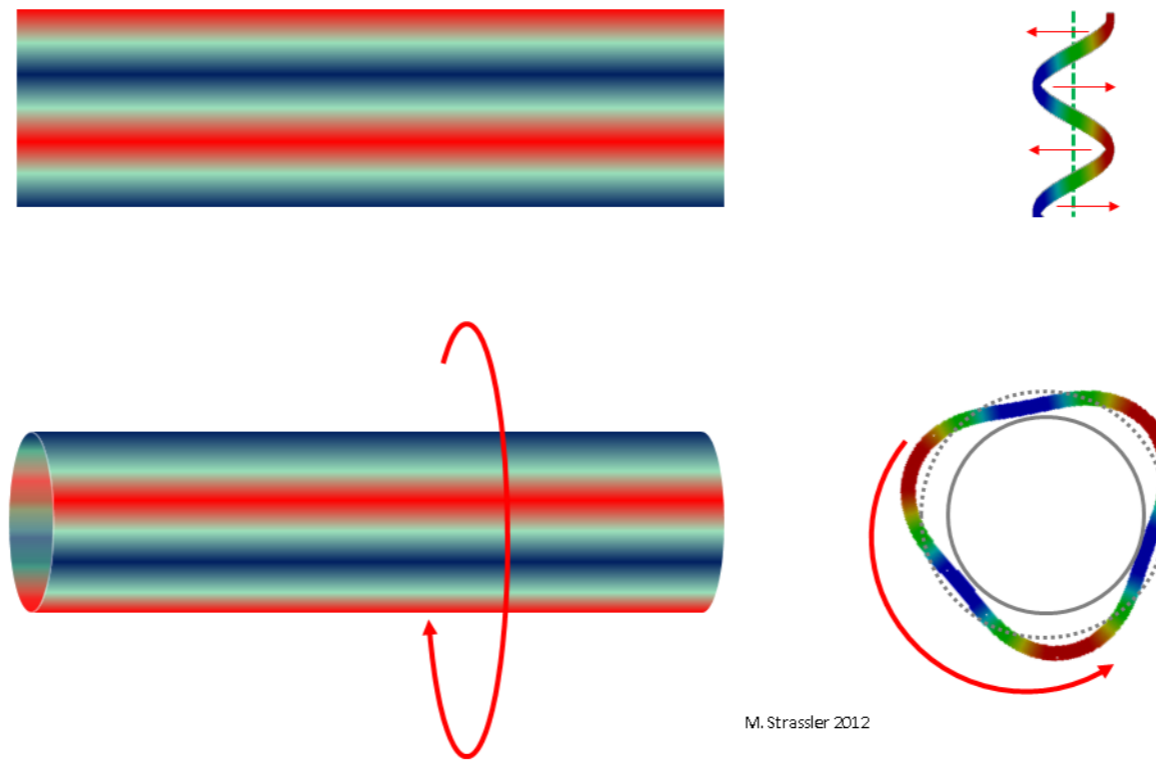
- * We can interpret one of these as parametrising the “radius” of the circle above each point in space.
- * The other three have precisely the right mathematical structure to give rise to an electromagnetic field in the lower dimensional theory!

$$\hat{g}_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} g_{\mu\nu} - \phi A_{\mu}A_{\nu} & -\phi A_{\mu} \\ -\sigma A_{\nu} & -\phi \end{pmatrix}$$

In such a theory, the 4d Coulomb interaction between charged particles will be modified. The 4d photon arises from the “zero momentum mode” of the extra metric components in the 5th circle.

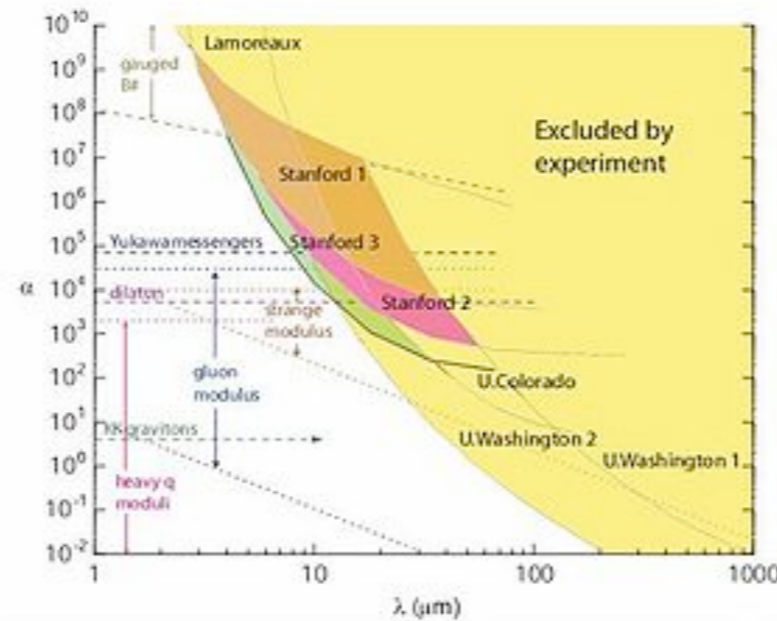
But there are also excited “Kaluza-Klein” cousins of the photon, whose momentum on the circle makes them massive in 4d:

Waves on the Strip and Tube with the Third-Longest Wavelength



Exchange of these “massive photons” would modify the force between charges in the real world.

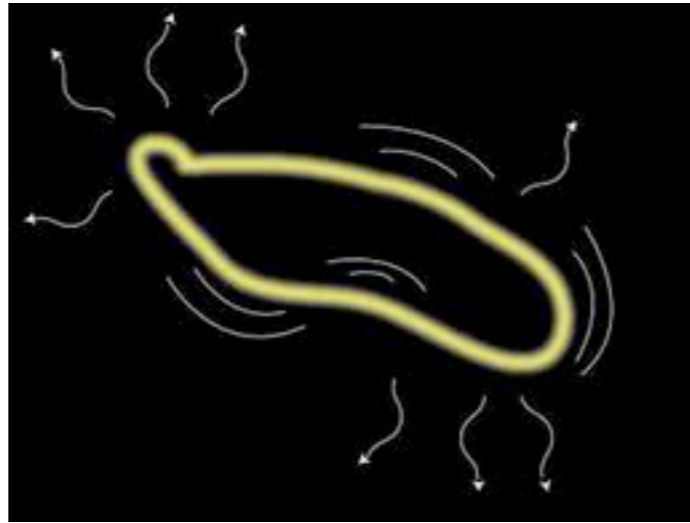
Experiment has not revealed any such modifications as
yet:



Modern incarnation of extra dimensions

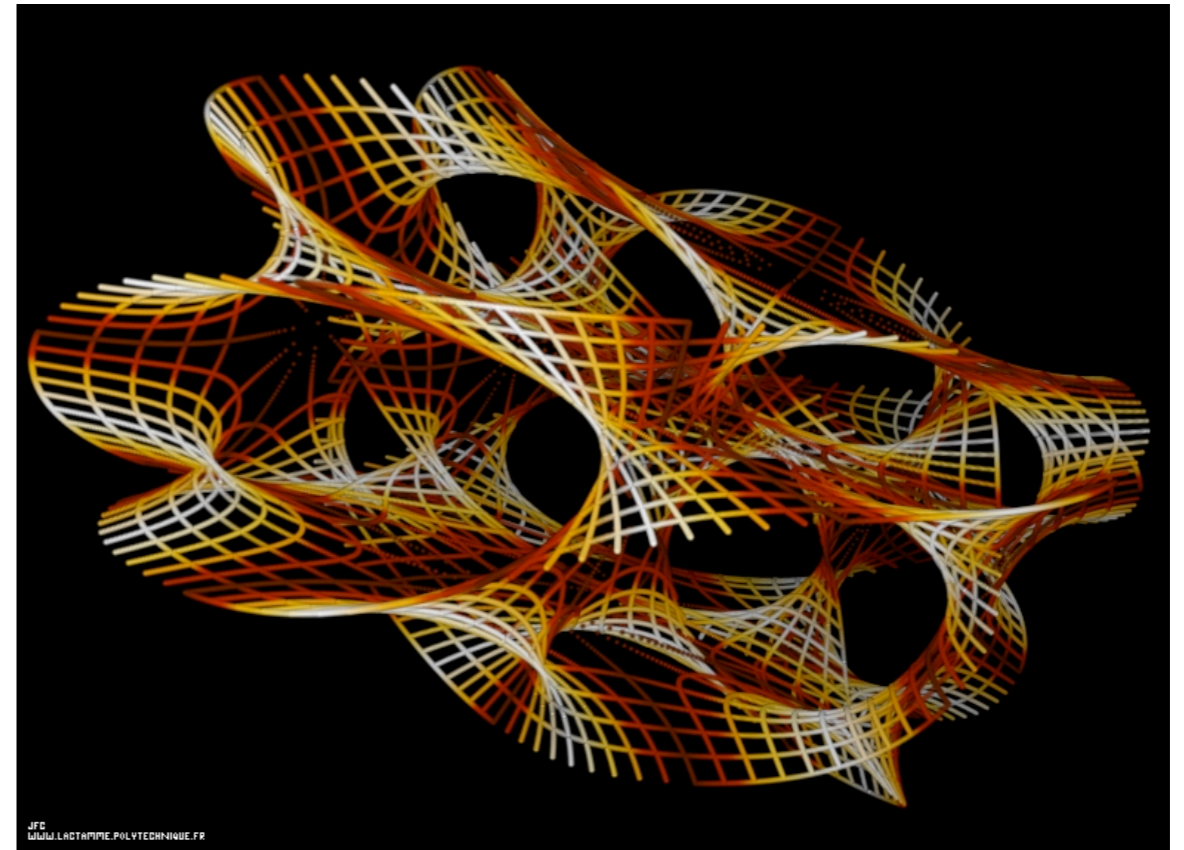
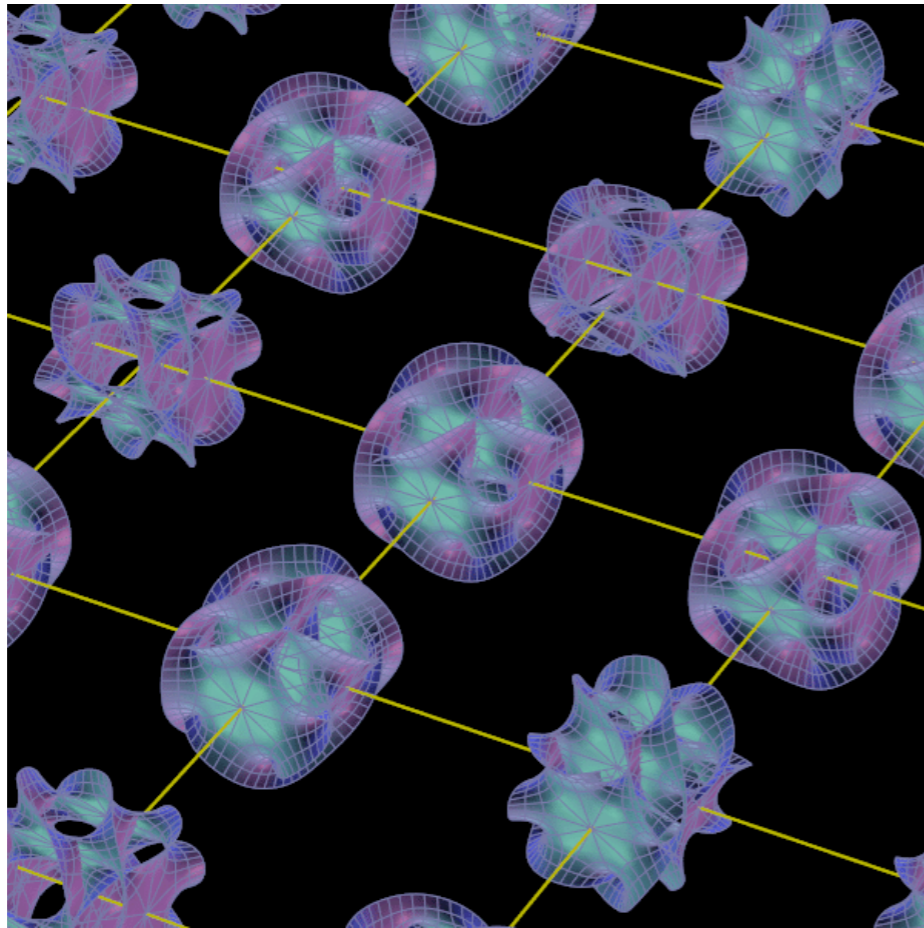
Since the time of Kaluza and Klein, we've learned about more fundamental interactions, and many new particles.

Our most promising way of making Einstein's theory consistent with quantum mechanics, replaces elementary particles with vibrating "superstrings":



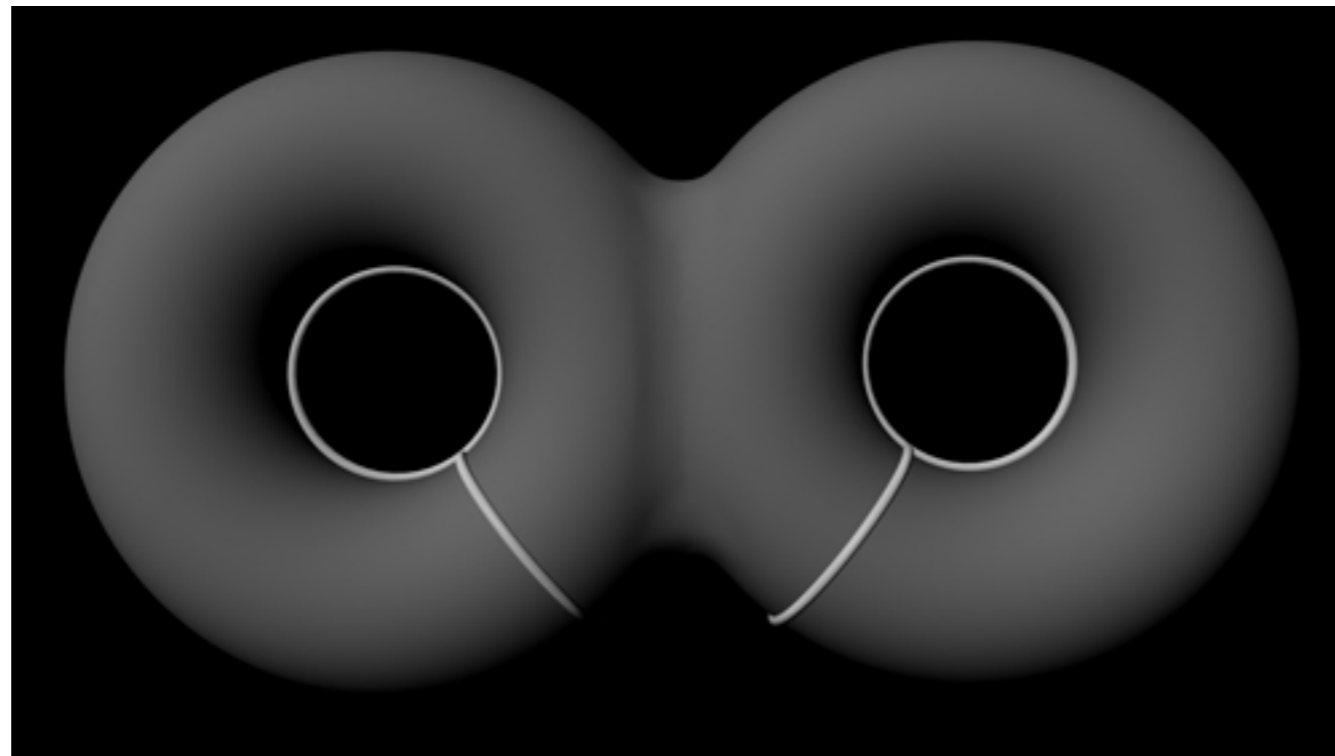
Mathematical consistency of these theories requires the existence of extra space-time dimensions; 6 new spatial dimensions are preferred in the simplest constructions.

The result is a rich generalization of the original Kaluza-Klein picture, where above each point in our space lies a six-dimensional “Calabi-Yau” manifold:



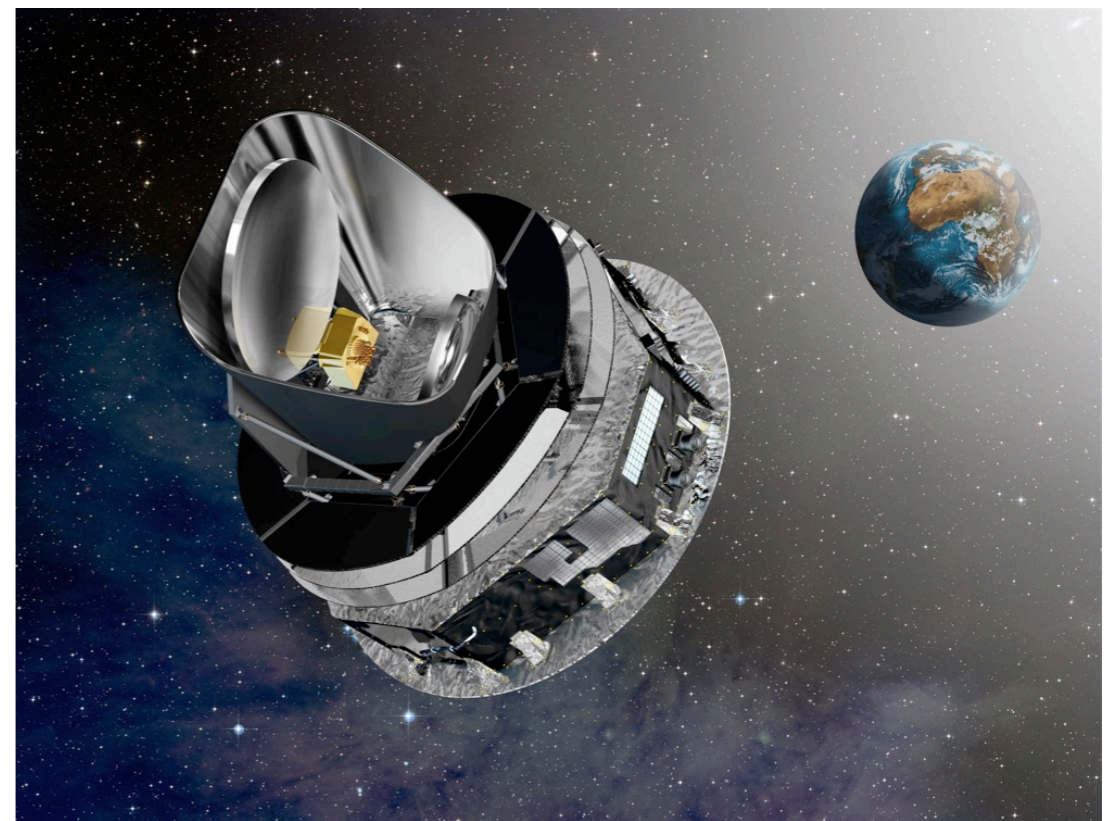
Thousands of possible such spaces with distinct topology are now known. They are rather beautiful.

The richer spectrum of extra dimensions and topologies gives string theorists the opportunity to explain the richer set of currently known forces and “elementary” particles, in terms of wrapped strings and branes:



Wrapping topologically distinct cycles yields different flavors of lower-dimensional particles.

We do not know whether this geometrization of physics via extra dimensions is true of our world or not. But we aim to find out!



Thanks for your attention!

Shamit Kachru
Stanford physics department
& SLAC theory group

[http://www.stanford.edu/dept/physics/people/faculty/
kachru_shamit.html](http://www.stanford.edu/dept/physics/people/faculty/kachru_shamit.html)